##### Chapter-2

**System of Linear Equations (SLE)**

In mathematics, linear systems are the basis and a fundamental part of [linear algebra](https://en.wikipedia.org/wiki/Linear_algebra), a subject which is used in most parts of modern mathematics. Computational [algorithms](https://en.wikipedia.org/wiki/Algorithm) for finding the solutions are an important part of [numerical linear algebra](https://en.wikipedia.org/wiki/Numerical_linear_algebra), and play a prominent role in [engineering](https://en.wikipedia.org/wiki/Engineering), [physics](https://en.wikipedia.org/wiki/Physics), [chemistry](https://en.wikipedia.org/wiki/Chemistry), [computer science](https://en.wikipedia.org/wiki/Computer_science), and [economics](https://en.wikipedia.org/wiki/Economics). A system of linear equations is a group of two or more linear equations containing the same variables. In a system of equations there is more than one unknown since the equations contain more than one variable. We will explore applications that involve systems of linear equations and look at how to set up a system of equations with given information. Systems of linear equations arise in a wide variety of applications. System of linear equations arises in the problem of polynomial curve fitting, network analysis and analysis of an electric circuit and the linear programming problem etc. System of linear equations also arises when we want to solve mixture problems and distance-rate-time problem. One of the most frequent occasions where linear systems of  equations in   unknowns arise is in least-squares optimization problems.  Least squares problems lead to square (i.e.) linear systems of equations. Also systems of linear equations arise in the problem of graph theory and cryptography. In cryptanalysis (breaking codes mathematically) we use linear in solving systems of equations related to both a grammar and language in cipher text.

**Linear equation:**

An equation in two or more variables (unknowns) is linear if it contains no products of unknowns or exponent of each unknown is .

**Example:**



**Solution:**

A solution of linear equation is a sequence of numbers  such that the equation is satisfied when we substitute. The set of all such solutions of the linear equation is called a solution set.



**System of linear equations:**

A group of linear equations of variables are of the form



is known as system of linear equation. Here the co-efficient  of the variable and the free term  are real numbers.

By a solution (set) of a system means such a set of real numbers that satisfies each equation in a system.

**Solution of a system of linear equations:**

A sequence of numbers is called solution of the system of linear equations given by if  is a solution of every equation in the system.

**Degenerate and non-degenerate linear equation:**

A linear equation is said to be **degenerate** if it has the form . That is, if every coefficient of the variable is equal to zero. The solution of such a generate linear equation is as follows:

(i) If the constant, then the above equation has no solution.

(ii) If the constant , then every vector  is a solution of the above equation.

The general linear equation  is called **non-degenerate** linear equation.

**Consistent and inconsistent equations:**

A system of linear equations is called consistent if it has at least one set of solution. A system of linear equations is called inconsistent if it has no solution.

**Consistency theorem:** The system of linear equations ( equations and unknowns) is consistent (i.e. there is at least one solution of the system) if the coefficient matrix and the augmented matrix have the same rank.

**Determinate and Indeterminate:**

A consistent system is called determinate if it has a unique solution and indeterminate if it has more than one solution.

An indeterminate system of linear equation always has an infinite number of solutions.

**Then 3 cases arise:**

► SLE is inconsistent  straight lines do not intersect (i.e., parallel);

► SLE has a unique solution  all straight lines pass through a single point;

► SLE is redundant  actually one straight line, with which others coincide, exists.

*y* *y* *y*

*x* *x* *x*

An inconsistent system A unique system Infinitely many solution system

(no common point) (only common point) (overlapping lines)

**Example:** Following augmented matrices illustrate the consistency of the linear system.

|  |  |  |
| --- | --- | --- |
| **(i)** ,  So, this system is consistent. | **(ii)** ,    So, this system is inconsistent. There is no solution for this system. | **(iii)** .  So, this system is consistent but infinitely many solutions. |

**Example:** Test the consistency of the following system of linear equations with the help of the rank of the matrix

.

**Solution:** The corresponding augmented matrix is

Now . Since , the system is inconsistent. It has no solution.

**Example:** Test the consistency of the following system of linear equations with the help of the rank of the matrix, If consistent solve the system.

.

**Solution:** The corresponding augmented matrix is

Now . Since , the system is consistent.

Echelon matrix can be written to system of linear equations

**Example: A system of linear equations with exactly one solution**

Consider the system





Solving the first equation for in terms of, we obtain the equation



Substituting this expression for into the second equation yields









Finally, substituting this value of into the expression for gives 

**Graphical representation of the system of linear equations for unique solution (One solution):**

C:\Users\Administrator\Desktop\Smart draw Figures\linear1.tif

Therefore, the unique solution of the system is given by and  Geometrically, the two lines represented by the two equations that make up the system intersect at the point  So, the solutions are and .

**Example:** **A system of linear equations which are coincident has infinitely many solutions:**

(Graphical representation of the system for infinitely many solutions)

Consider the system ; .

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Solving the first equation for in terms of, we obtain the equation



Substituting this expression forinto the second equation yields







Which is a true statement. This result follows from the fact that the second equation is equivalent to the first. Our computations have revealed that the system of two equations is equivalent to the single equation . Thus, any ordered pair of numbers  satisfying the equations  or constitutes a solution to the system.

In particular, by assigning the value  to , where  is any real number, we find that and so the ordered pair is a solution of the system. The variable  is called a parameter. For example, setting gives the point as a solution of the system, and setting gives the point as another solution. Since  represents any real number, there are infinitely many solutions of the system. Geometrically, the two equations in the system represent the same line, and all solutions of the system are points lying on the line (Figure). Such a system is said be dependent.

**Example: A system of linear equations that has no solution:**

Consider the system





Solving the first equation for in terms of, we obtain the equation



Substituting this expression for into the second equation yields



which is clearly untrue. Thus, there is no solution to the system of equations.





**Graphical representation of the system of linear equations for no solution:**

C:\Users\Administrator\Desktop\Smart draw Figures\linear3.tif  
It has been observed that these two lines are parallel to each other.

**Homogeneous and nonhomogeneous linear equation:**

A system of linear equations is called homogeneous if all the constant terms of the Non-homogeneous system are zero such as the system has the form:



Homogeneous system of linear equations has two types of solutions. They are

1. Trivial (zero) solution (all )
2. More than one solutions

(i)

(ii) *y*

*x*

Trivial (zero solution) More than one solutions

**Matrices and system of linear equations:**

The system of linear equations (1)can be written in the matrix form.



or simply …………………………………….…(2)

where co-efficient matrix,, variable matrix,and constant matrix,

The associated homogeneous system of (1) is .

The system (1) also can be written in augmented matrix form or .

There are three commonly used methods to solve system of linear equations:

1. Using inverse matrix,
2. Using elementary row operations (Gaussian elimination and Gauss-Jordan elimination),
3. Cramer’s rule.

**Example:** Given that

The augmented matrix of the above system of linear equations is

.

**Solution of linear equation by applying matrices:**

**for the system of linear equations:**

Consider, for the system of linear equations (2).

Let, be the determinant of the matrix . we have to evaluate the determinant. If , is singular. So  doesn’t exist and hence the system has no solution. If , A is nonsingular. So,  exists and hence the system has a solution. Now multiplying both sides of (3) by , we have

since  and .

That is,

 (say)

Where determinant of the matrix, *A* is . Then,  (say) is a solution of the given system of *n* linear equations.

It is to be noted that the solution of the system of equation can also be found by reducing the augmented matrix of the given system to reduced echelon form.

**(no. of linear equations) (no. of unknowns or variables) of the following system of linear equations:**

After reduced the system of linear equations (1) into echelon form,

1. Number of variable(s) is equal to the number of equation(s) gives the unique solution
2. Number of variable(s) is greater than the number of equation(s) gives more than one solution.

**Example of the algorithm**

Suppose the goal is to find and describe the set of solutions to the following [system of linear equations](http://en.wikipedia.org/wiki/System_of_linear_equations):

The table below is the row reduction process applied simultaneously to the system of equations, and its associated [augmented matrix](http://en.wikipedia.org/wiki/Augmented_matrix). The row reduction procedure may be summarized as follows: eliminate *x* from all equations below , and then eliminate *y* from all equations below . This will put the system into [triangular form](http://en.wikipedia.org/wiki/Triangular_form). Then, using back-substitution, each unknown can be solved.

|  |  |  |
| --- | --- | --- |
| System of equations | row operations | augmented matrix |
|  |  |  |
|  |  |  |
|  |  |  |
| The matrix is now in echelon form | | |
|  |  |  |
|  |  |  |
|  |  |  |

The solution is , , and . So, there is a unique solution to the original system of equations.

**Example:** Solve the following system of equations using Gaussian elimination method

|  |  |
| --- | --- |
|  |  |
|  |  |
| From this stage, we can get the solution by back solving | and |
| So, the solution is: |  |

**Example:** Solve the following system of equations using elementary row operations

|  |  |
| --- | --- |
|  |  |
|  |  |

Let, , where is a free variable.

We have,

So, the general solution of the system is .

For particular solution, putting

**Example:** Solve the following system of equations using Gaussian elimination method

|  |  |
| --- | --- |
|  |  |
|  |  |

The third row it does not exist. So, the system is inconsistent. That means the system has no solution.

**Example:** Solve the following system using Gauss-Jordan elimination method.

**Solution**

Hence the solution is .

**Example:** Solve the following system of equations using matrix inversion and justify your answer.

(

**Solution:**

System (1) is written in the matrix form

Where, , , and

The determinant of the matrix *A* is

So, the matrix is non-singular and exists.

Now

And

Verification:

**Example:** Solve the following system of equations using matrix inversion.

**Solution:** We write down the given system as



  [ say]

 [ since exists]

 =  = [ after finding ]

 is the required solution.

**Example**: Using matrix inversion solve the system of linear equations

.

**Solution:**

The system of equations can be written in the matrix form as , where , and . The solution can be written as . Let us find using elementary row operations.

Now, .

**Cramer’s Rule:**

Let a system of linear equations is given . This system is

**(a)** inconsistent if but ;

**(b)** consistent and redundant if and ;

**(c)** consistent and unique if ; and in this case the solution is given by

,

can be obtained by replacing ith column by right hand side.

### Explicit formulas for small systems

Consider the linear system

which in matrix format is

Assume  nonzero. Then, with help of determinants and  can be found with Cramer's rule as

The rules for matrices are similar. Given

 which in matrix format is

Then the values of , and can be found as follows:

Similar idea can be extended for systems

**Example:**

Let us demonstrate Cramer's rule with the following system:

**Step 1:**

The coefficient matrix of this system is

Note that the matrix is square (it has rows and columns), and so we may proceed with the next step of Cramer's rule.

**Step 2:**

Now find the determinant of the coefficient matrix ; use the matrix manipulator in the tools box if you would like help in this computation. You should get . This is not zero, so Cramer's rule may be applied here.

**Step 3:**

and its determinant is . Therefore

**Step 4:**

Using the same method, the values for the remaining variables, and , are computed below:

and its determinant is . Therefore

 and its determinant is . Therefore

**Example:** Verify whether the following system of linear equations is consistent or not. If consistent then solve the system using Cramer’s rule. Also check your answer.

.

**Solution:**

= 0 and . Therefore, this SLE is either inconsistent or more than one solutions.

**Example:**

**Solution:**

and and, therefore, this SLE has a unique solution.

Now  ; ; .

; ;

Thus  ;  ; .

**Example:**

Solve the following system of linear equations using Cramer’s rule

We have the left-hand side of the system with the variables (the "coefficient matrix") and the right-hand side with the answer values. Let be the determinant of the coefficient matrix of the above system, and let  be the determinant formed by replacing the -column values with the answer-column values: Evaluating each determinant, we get:

So, according to Cramer's rule:

**Example:**

Solve the following system of linear equations using Crammer’s rule

.

**Solution:** Each unknown will be the quotient of the determinant obtained by substituting the answers in the right sides of the equations for the coefficients of the unknown divided by the determinant formed by taking the coefficients on the left sides of the equations.

, ,

**Example:**

Determine the value(s) of andsuch that the following system of linear equations has no solution, more than one solution, and a unique solution.



**Solution:** The corresponding augmented matrix is

The above system is in echelon form. Now we consider the following three cases:

1. If and then third equation of is of the form , where which is not true. So, the system is inconsistent. Thus, the system has no solution for and .
2. If and then third equation of is vanishes and the system will be in echelon form having two equations in three variables. So, it has free variables which is . Hence the given system has more than one solution for and .
3. For a unique solution, the coefficient of in the 3rd equation must be non-zero i.e., and may have any value. Therefore, the given system has unique solution for and arbitrary values of.

**Example:**

Determine the value(s) of and such that the following system of linear equations has no solution more than one solution and a unique solution.

.

**Solution:** The given system of liner equations is 

Reduce the system to echelon form by means of elementary row operations,









The above system is in echelon form. Now we consider the following three cases:

1. From third equation of, we see that if or then the equation becomes, which is contradiction. Therefore, the system is inconsistent if. Thus, the system has no solution for.
2. We know, if the number of variables is greater than the number of equations, then the system has more than one solution. From third equation of, we see that if then it becomes  . In this case the system has three variables with two equations. So, the given system has more than one solution for.
3. We know, if the number of variables and the number of equations be equal, then the system has unique solution. The systemhas a unique solution



**Example:**

Determine the value(s) of and such that the following system of linear equations has no solution, more than one solution, and a unique solution.



**Solution:** The given system of liner equations is



Reduce the system to echelon form by means of elementary operations,





The above system is in echelon form. Now we consider the following three cases:

1. For a unique solution, the coefficient of in the 3rd equation ofmust be non-zero i.e., andmay have many values. Therefore, the given system has unique solution for  and arbitrary values of .
2. If and then third equation of is vanishes and the system will be in echelon form having two equations in three variables. So, it has free variables which is . Hence the given system has more than one solution for and .
3. If and then third equation of is of the form , where which is not true. So, the system is inconsistent. Thus, the system has no solution for and .

**Example:**

Find the values of such that the following system of linear equations has non-zero solution.

.

**Solution:**

The augmented matrix

~

On interchanging first row and third row, we have

~

Reducing the system to row echelon form by the elementary row operations …

~

~

So,

**Example:** A medicine company wishes to produce three types of medicine : type and . To manufacture a type medicine requires minutes each on machine and and minutes on machine . A type of medicine requires minutes on machine , minutes on machine and minutes of machine . A type medicine requires minutes on machine, minutes on machine and minutes on machine . There are hours available on machine hours available on machine and hours available on machine. How many medicine of each type should company make in order to use all the available time?

**Solution:**

Here, hours minutes, hours minutes and hours minutes.

Let , and be the number of medicines of types , and respectively. Then we have the following system of linear equations:

The augmented matrix of the above system is

Reducing the system to echelon form by the elementary row operations

Hence the solution of the above system is

Thus, the number of each type of medicine is .

**Example:** Determine the polynomial whose graph passes through the points , and 

**Solution:**

Given polynomial

Substituting and into  and the corresponding values produces the system of linear equations in the variables and shown below:



Reducing this system to echelon form by the elementary operations,





By back substitution method from 3rd equation, we have 

From the 2nd equation, we get 

and from 1st equation, we get 

Hence, the solution of this system is ,and .

So, the polynomial function is 

The graph is shown in the following figure:

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**Example:** Find the polynomial that fits the points and .

**Solution:** We have provided five points, so we chose a fourth-degree polynomial function

Substitution the given points into products the system of linear equations listed below:

The solution of the above system is

Which means the polynomial function is

The graph of is shown in the following figure:

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**Example:**

Set up a system of linear equations to represent the network shown in the following figure and solve the system.

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**Solution:**

Each of the network’s five junctions gives rise to a linear equation, as shown below:

The augmented matrix is



Reduce the system to echelon form by the elementary row operations















The corresponding system of equations are as follows:



The above system is in echelon form having equations in unknowns. So, it has free variable, which is 

Let then by back substitution method, we have

where  is a real number.

So, this system has an infinite number of solutions.

**Example:**

Determine the currents and for the electrical network shown in the following figure.

|  |  |
| --- | --- |
| **Solution:**  Applying Kirchhoff’s current law to each junction produces  ; Junction or junction  and applying Kirchhoff’s second law to two paths produces    So, we have the following system of three linear equations in the and | C:\Users\Administrator\Desktop\circuit2.tif |

The augmented matrix of the above system is



Reducing the system to echelon form by the elementary row operations













So, the solution of the above system is and 

C:\Users\Administrator\Desktop\circuit3.tif

**Example:**

Determine the currents  and for the electrical network shown in the following figure.

**Solution:**

Applying Kirchhoff’s current law to each junction produces

 junction

 junction

 junction

 junction

and applying Kirchhoff’s second law to the three paths produces



Now we have the following system of seven linear equations in the variables and 



Using Gauss-Jordan elimination method, we have solution of the above system is

and 

which means andand 

**Example:**

Determine the loop currents and of the following circuit using mesh analysis.

|  |  |
| --- | --- |
| **Solution:**  For loop 1:  For loop 2:  Thus, the system of linear equations is  Therefore, | C:\Users\Administrator\Desktop\Smart draw Figures\circuit6.tif |

**Example:**

Determine the loop currents ,  and of the following circuit using mesh analysis.

|  |  |
| --- | --- |
| **Solution:**  For loop :  For loop :  For loop :  Thus, the system of linear equation is | C:\Users\Administrator\Desktop\Smart draw Figures\circuit7.tif |

Hence solving the system,

**Example:**

The following figure shows the flow of downtown traffic in a current city during the rush hours on a typical weekday. The arrows indicate the direction of traffic flow on each-way road, and the average number of vehicles per hour entering and leaving each intersection appears beside each road. 5th Avenue and 6th Avenue can each handle up to vehicles per hour without causing congestion, whereas the maximum capacity of both 4th street and 5th street is vehicles per hour. The flow of traffic is controlled by traffic lights installed at each of the four intersections.

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1. Write a general expression involving the rates of flow and suggest two possible flow patterns that will ensure no traffic congestion.
2. Suppose the part of 4th street between 5th Avenue and 6th Avenue is to be resurfaced and that traffic flow between the two junctions must therefore be reduced to at most vehicles per hour. Find two possible flow patterns that will result in a smooth flow of traffic.

**Solution:**

1. To avoid congestion, all traffic entering an intersection must also leave that intersection. Applying this condition to each of the four intersections in a clockwise direction beginning with the 5th Avenue and 4th Street intersection, we obtain the following equations:









This system of four linear equations in the four variables may be written in the more standard form



Using Gauss-Jordan elimination method, we obtain









The last augmented matrix is in row-reduced form and is equivalent to a system of three linear equations in the four variables . This we may express three of the variables-say, in terms of . Setting (*t* a parameter), we may write the infinitely many solutions of the system as









Observe that for a meaningful solution we must have since must all be nonnegative and the maximum capacity of a street is .

For example, picking gives the flow pattern



Selecting gives the flow pattern



1. In this case, must not exceed . Again, using results of part, we find, upon setting the flow pattern obtained earlier.
2. Picking gives the flow pattern 

**Linear Programming Problem:**

The linear programming is the modern method of mathematics to solve the system of linear inequalities. The solution makes the objective linear function a minimum (or maximum) and which satisfies the constraints and non-negative conditions.

General linear programming problems:

Let be a linear function by

where is set of *n* constants.

Let be constants and be a set of constants such that

and finally let

1. The problem of solving the values of which make a minimum (or maximum) and which satisfies equations and is called general linear programming.
2. is called objective function.
3. System of linear inequalities eqn is called constrains and in eqn is called non negative restriction.
4. Solutions: Values of unknowns which the constraints eqn of a general linear programming problem are called general solutions.
5. Feasible solution: Any solution if GLPP which satisfies the non-negative restrictions of the problem is called feasible solution of GLPP.
6. Optimum solution: Any feasible solution which optimizes (minimizes, maximizes) the objective function is called optimum solution.

**Linear programming problem** can be solved by (i) Graphical method, (ii) Simplex method (Pivoting method).

**Example:** A company manufactures and sells two models of lamps, L1 and L2. To manufacture each lamp, the manual work involved in model L1 is 20 minutes and for L2, 30 minutes. The mechanical (machine) work involved for L1 is 20 minutes and for L2, 10 minutes. The manual work available per month is 100 hours and the machine is limited to only 80 hours per month. Knowing that the profit per unit is 15 and 10 for L1 and L2, respectively, determine the quantities of each lamp that should be manufactured to obtain the maximum benefit.

Solution:

Let

= number of lamps L1

= number of lamps L2

Objective function

Convert the time from minutes to hours.

20 min = 1/3 h 30 min = 1/2 h 10 min = 1/6 h

|  |  |  |  |
| --- | --- | --- | --- |
|  | L1 | L2 | Time |
| Manual | 1/3 | 1/2 | 100 |
| Machine | 1/3 | 1/6 | 80 |

Writing the [constraints](https://www.superprof.co.uk/resources/academic/maths/linear-algebra/linear-programming/linear-programming.html#co) as a [system of inequalities](https://www.superprof.co.uk/resources/academic/maths/algebra/inequalities/systems-of-inequalities.html) we get

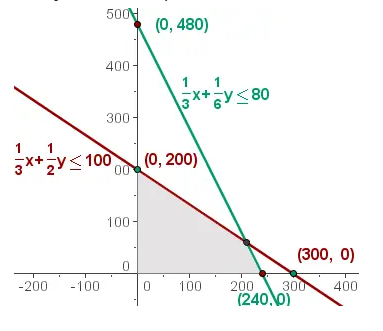
As the numbers of lamps are natural numbers, we have

Represent the constraints graphically.

As , work in the first quadrant.

Solve the inequation graphically: ; and take a point on the plane, for example (0,0).

The area of intersection of the solutions of the inequalities would be the solution to the system of inequalities, which is the set of feasible solutions.



The optimal solution, if unique, is a vertex. These are the solutions to systems:

To determine which of them has the maximum or minimum values.

In the objective function, place each of the vertices that were determined in the previous step.

So, (210,60) is our required answer.

**Example:** A calculator company manufactures two types of calculator: a handheld calculator and a scientific calculator. Statistical data projects that there is an expected demand of at least 100 scientific and 80 handheld calculators each day. Since the company has certain limitations on the production capacity, the company can only manufacture 200 scientific and 170 handheld calculators per day. The company has received a contract to deliver a minimum of 200 calculators per day. If there is a loss of 2 taka on each scientific calculator that you sold and a profit of 5 taka on each handheld calculator, then how many calculators of each type the company should manufacture daily to maximize the net profit?

**Solution:** To solve this problem, let’s first formulate it properly by following the steps.

**Step 1: Identify the number of decision variables.**

In this problem, since we have to calculate how many calculators of each type should be manufactured daily to maximize the net profit, the number of scientific and handheld calculators each are our decision variables.

Consider,

number of scientific calculators manufactured =

number of handheld calculators manufactured =

**Step 2: Identify the constraints on the decision variables**.

The lower bound, as mentioned in the problem (there is an expected demand of at least 100 scientific and 80 handheld calculators each day) are as follows.

Hence, and

The upper bound owing to the limitations mentioned the problem statement (the company can only manufacture 200 scientific and 170 handheld calculators per day) are as follows:

Hence, and

In the problem statement, we can also see that there is a joint constraint on the values of and due to the minimum order on a shipping consignment that can be written as:

**Step 3: Write the objective function in the form of a linear equation.**

In this problem, it is clearly stated that we have to optimize the net profit. As stated in the problem(If there is a loss of 2 taka on each scientific calculator that you sold and a profit of 5 taka on each handheld calculator), the net profit function can be written as:

**Profit (P) =**

**Step 4: Explicitly state the non-negativity restriction.**

Since the calculator company cannot manufacture a negative number of calculators.

**Hence, and**

Since we have formulated the problem, let’s convert the problem into a mathematical form to solve it further.

Maximization of P =

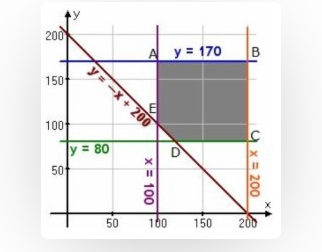
subject to:

**Step 5: Plot the constraints on the graph.**

Let’s plot all the constraints defined in step 2 on a graph in a similar manner as we plot an equation.

**Step 6: Highlight the feasible region on the graph.**

After plotting the coordinates on the graph, shade the area that is outside the constraint limits (which is not possible). The highlighted feasible area will look like this:



**Step 7: Find the coordinates of the optimum point.**

To find the coordinates of the optimum point, we will solve the simultaneous pair of linear equations.

**Corner Points Equation, P =**

**A (100, 170) P = 650**

**B (200, 170) P = 450**

**C (200, 80) P = 0**

**D (120, 80) P = 160**

**E (100, 100) P = 300**

**Step 8: Find the optimum point.**

The above table shows that the maximum value of P is **650** that is obtained at

**(, ) = A (100, 170).**

**Cryptographically Problem:**

The process to write (encoded) and read (decoded) any secret messages by using matrices is known as Cryptography.

**Specific Aims:** We will

* **be able to encode a message using matrix multiplication;**
* **decode a coded message using the matrix inverse and matrix multiplication.**

**Algorithm to Encode a Message:**

* Assign the numbers 1-26 to the letters (capital/small) in the alphabet given below and assign the number 0 to a blank to provide for space between words.

Blank

**0**

A B C D E F G H I J K L M N

1 2 3 4 5 6 7 8 9 10 11 12 13 14

O P Q R S T U V W X Y Z

15 16 17 18 19 20 21 22 23 24 25 26

* Write the provided message corresponds to the sequence of numbers.
* Matrix, “A” (say) can be used as an encoding matrix, if elements are positive integer of the considered matrix and inverse matrix exists.
* Divide the numbers in the sequence into groups of the order of matrix, “A” (or size of the matrix) and use these groups as the columns of a matrix, “B” (say). Proceed down the columns not across the rows.
* Write the provided message corresponds to the sequence. Then, multiply this matrix, “B” on the left by matrix, “A”.
* Coded message will be written by picking the elements in each column from left of the matrix “AB”.

**Algorithm to Decode a Message:**

* Find the inverse of encoding matrix, “A”, if exists.
* Divide the numbers in the sequence into groups of the order of matrix, “A” (or size of the matrix) and use these groups as the columns of a matrix, “B” (say). Proceed down the columns not across the rows.
* Multiply this matrix, “B” on the left by inverse matrix, “”.
* Writing the numbers in the columns of this matrix “” in sequence and using the letters to correspondence numbers given below.

O P Q R S T U V W X Y Z

15 16 17 18 19 20 21 22 23 24 25 26

Blank

**0**

A B C D E F G H I J K L M N

1 2 3 4 5 6 7 8 9 10 11 12 13 14

* These letters give decoded message.

**Example:** Encode the message **SECRET CODE** by using matrix .

**Solution:**

**Step 1:** The provided message “**SECRET CODE** ” corresponds to the sequence

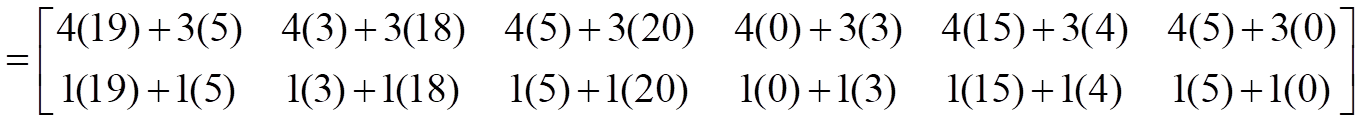
**S E C R E T space C O D E**

**19 5 3 18 5 20 0 3 15 4 5**

**Step 2:** Divide these numbers in the sequence into groups of 2 (based on the size of given matrix) and use these groups as the columns (proceed down the columns) of a matrix, B of two rows. Thus,



**Step 3:** Now,





**Step 4:** Therefore, the coded message is **91 24 66 21 80 25 9 3 72 19 20 5.**

**Example:** The encoded message is 7 **6 28 20 23 5**. Decode this message by using matrix, .

**Solution:**

**Step 1:** When elements of the encoding matrix, are positive and find inverse matrix of .

**Step 2:** The encoded message is 7 **6 28 20 23 5.**

Since the encoding matrix, is , make a matrix “C” having two rows by picking two numbers from the left of encoded message as columns of matrix “C”. We have,

**Step 3:** Now,

**Step 4:** Writing the numbers in the columns of this matrix in sequence and using the letters to correspondence numbers. Thus,

**6 1 20 8 5 18**

**F A T H E R**

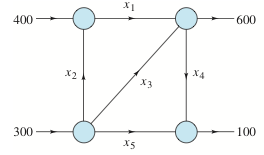
**Exercise 2**

1. Test whether the systems of linear equations are consistent. If consistent find the solutions of the system. Also check your answer by direct substitution.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **b.**  Ans: Inconsistent | b. | Ans: Inconsistent | **c.** | Ans. |
|  |  |  |  |  |  |
| **d** | Ans: | **e.** |  | **f.** | Ans: |
|  |  |  |  |  |  |
| **g.** | Ans. | **h.** | Ans. | **i.** | Ans: |
|  |  |  |  |  |  |
| **j.** | Ans: | **k.** |  | **l.** | Ans: |
|  |  |  |  |  |  |
| **m.** | Ans: | **n.** | Ans: | **o.** | Ans: |
|  |  |  |  |  |  |
| **p.** | Ans: | **q.** | Ans: | **r.** | Ans: |
|  |  |  |  |  |  |
| **s.** | Ans: | **t.** | Ans: | **u.** | Ans: |

1. The network in the figure below shows the traffic flow (in vehicles per hour) over the

several one-way streets. Determine the general flow pattern for the network.



Ans: Here are free variables.

1. The network in the figure below shows the traffic flow (in vehicles per hour) over the

several one-way streets. Determine the general flow pattern for the network.

C:\Users\Administrator\Desktop\traffic.tif

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|  |  |
| --- | --- |
| **a.** | **b.** |
| C:\Users\Administrator\Desktop\traffic3.tif | C:\Users\Administrator\Desktop\Smart draw Figures\traffic4.tif |

Ans: (a) Here is the free variable. The system has an infinite number of solutions. But to remove negativity must be between

(b) Here is the free variable. The system has an infinite number of solutions. But to remove negativity must be greater than

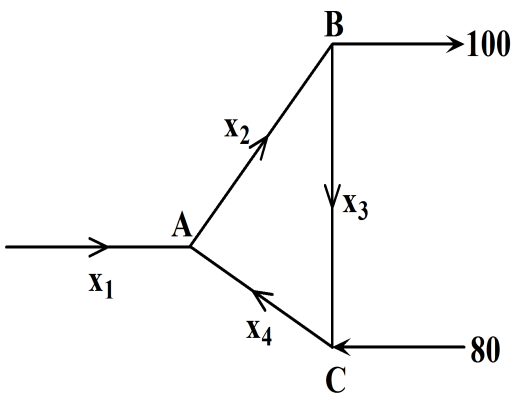
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several one-way streets. Determine the general flow pattern for the network.

C:\Users\Administrator\Desktop\Smart draw Figures\ntwork2.tif

Ans: Here is the free variable. The system has an infinite number of solutions. But to remove negativity must start from

1. Find the general flow pattern of the network system in the figure. Assuming that the flows are all nonnegative, what is the smallest possible value for.

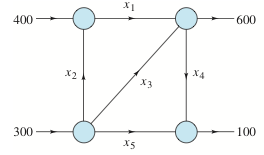


1. A company produces three products every day. Their total production on a certain day is tons. It is found that the production of exceeds the production of by tons while the total production of and is twice the production of . Determine the production level of each product.

Ans:

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|  |  |
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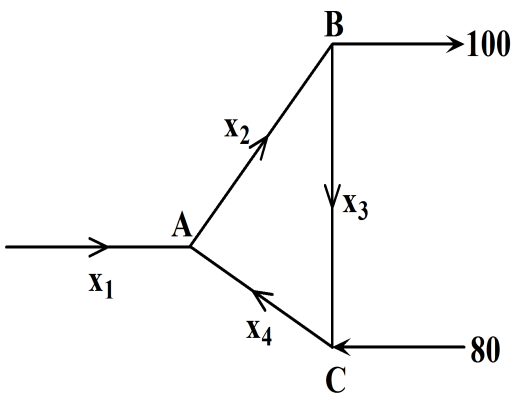
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Ans:

1. Construct the system of linear equations from the following diagrams, reduced the system to echelon form and finally find the general flow pattern, where is the number of cars.

(i)

Shape

Description automatically generated with medium confidence

(ii)

A black background with a black square

Description automatically generated with medium confidence

(iii)

Shape

Description automatically generated with medium confidence

(iv)

Shape

Description automatically generated with medium confidence

1. Encode the message **TRY YOUR BEST** by using matrix, . Also decode your encrypted message using the same matrix**.**
2. Encode the message **HONESTY IS THE BEST POLICY** by using matrix given above.
3. Encrypt the message **EFFORT NEVER DIES** by the provided matrix, .
4. The encoded message is  **28 13 28 20 23 5**. Decode this message by using matrix, .

**Linear Programming:**

**Reference: Operation Research by Hamdy A. Taha, 10th edition.**

**Problem-2.1-1 and 2.1-2 (Page-45-52), problem 2.7 , 2.8, 2.9, 2.13 (page 77, 78).**

**Supplementary:**

MATLab command for finding unique solution (if exists) of a system of equation:



Ans:

|  |  |  |
| --- | --- | --- |
| For | Input Command | Output |
| Coefficient matrix: | >> A = [3 2 -1;2 3 -3;1 -1 6] | A =  3 2 -1  2 3 -3  1 -1 6 |
| Right hand side matrix: | >> B= [20;7;41] | B =  20  7  41 |
| checking whether there exists a unique solution or not! | >> if det(A)~=0  disp ('There exists a unique solution for the given system.')  else  disp ('There is no unique solution for the given system.')  end | There exists a unique solution for the given system. |
| Solution set, where, | >> X=inv(A)\*B | X =  5.0000  6.0000  7.0000 |

References:

* Linear Programming by Thomas S. Ferguson
* Linear Programming by George B. Dantzig, Mukund N. Thapa
* Operations Research by Ravindran, Phillips & Solberg
* Operations Research by H. Taha